The Operational Amplifier

4.1 Introduction

Operational amplifiers are one of the most extensively used analog integrated circuits especially because of their ability to approximate reasonably well the ideal behavior. For this reason real operational amplifiers can be quite often modeled as ideal. This simplicity of usage and mostly the versatility this device, which hides a large internal complexity [1], make the operational amplifier suitable for many different applications.
The ideal operational amplifier concept introduced in the first section, will look after a first reading quite awkward. Anyway, everything will be clearer when it is analyzed in conjunction with the called feedback network which links the output of the amplifier inputs.
Subsequent sections are mainly dedicated to the explanation of some basic circuits. Finally, section 4.4 introduces more realistic model of operational amplifier together with some of the peculiar behavior of this electronic device.

4.2 The Ideal Operational Amplifier

The ideal operational amplifier (Op-Amp) is a linear amplifier with two differential inputs v+, v- and one output v_o (see figure 4.1) and with the following characteristics:

- $v_0 = A_v(v^+ - v^-)$, $A_v > 0$, (linearity)
- input resistance $R_i \rightarrow \infty$,
- output resistance $R_o \rightarrow 0$,
- voltage gain $A_v \rightarrow 1$,
- frequency response constant for any frequency.

Aside the welcome property of linearity and infinite frequency response, the need of all the other characteristics can be justified as follows. Infinite input resistance R_i means essentially that the Op-Amp inputs do not produce perturbations to any circuit to which they are connected to. Zero output resistance R_o perfectly isolates the Op-Amp from any perturbation. Infinite input impedance and zero output impedance implies also no dissipation of energy. The condition of infinite voltage gain A_v is necessary if we want a device able to deliver any gain, once a network which connects the output to the input is added to the Op-Amp. In general, this kind of network is called *feedback network.*

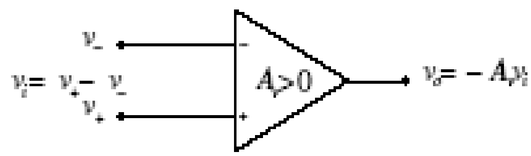

Figure 4.1: Op-Amp symbol.

4.2.1 Fundamental Equation for the Ideal Op-Amp (the Golden Rule)

The consequence of the following conditions

- $A_v \rightarrow \infty$,
- $v_o \rightarrow \infty$, if $v_i = v^+ - v^- < \infty$,

Equation 4.1 will be called the *Op-Amp golden rule* and is fundamental for the solution of any circuit involving Op-Amps. We will see in the next sections the importance of this equation once a feedback network is connected to the Op-Amp.

4.2.2 Op-Amp Input Output "Logic"

It is worthwhile here to notice the behavior of Op-Amp output as function of the two inputs. From the definition of Op-Amp we have that a signal sent to the negative input V_- is amplified and changed in sign. A signal sent to the positive input V_+ is just amplified. Two signals sent each to one input are indeed subtracted and amplified.

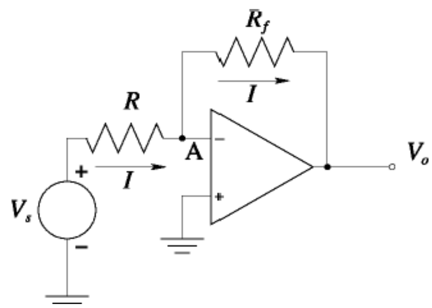

Figure 4.2: Op-Amp with a feedback network.

4.2.3 Op-Amp with a Feedback Network

Let's consider the circuit in figure 4.2, where a feedback resistance R_f is connected to the negative input. The current through the resistors R and R_f is the same because the ideal Op-Amp input does not drive any current ($R_i = \infty$,). Furthermore, since $V_i = V^+ - V^- = 0$ and with the use Ohm's law and the KVL, it follows that

$$I = \frac{V_i}{R} = -\frac{V_o}{R_f} \qquad 4.2$$

The output voltage V_o and voltage gain A will be

$$V_0 = AV_i, \qquad A = -\frac{R_f}{R}$$

The gain of the Op-Amp depends just on the resistances ratio R_f and R.

4.2.4 The Virtual Ground

Let's re-analyze the circuit in figure 4.2. Because of the golden rule $V i = V^+ - V^- = 0$, and the negative input is grounded, the node **A** must always be at zero voltage. This is equivalent to having **A** virtually grounded. The adjective virtual is necessary, otherwise we could not have $R_i = \infty$. In other words, the virtual ground happens to be because the Op-Amp does its best to keep $V_i = 0$.

4.3 Commonly Used Op-Amp Circuits

In the study of the several common Op-Amp configurations, we will use the approximation of an ideal circuit. A more realistic model is often necessary to understand some behaviors of real circuits. For an initial design, and where the the ideal Op-Amp characteristics are well approximated, the ideal model is quite often sufficient.

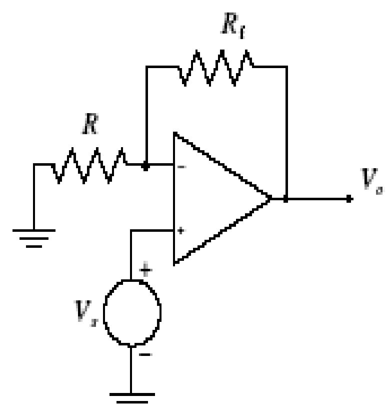

Figure 4.3: Non-inverting configurations of the Op-Amp.

4.3.1 Non-Inverting Amplifier

Let's consider the non-inverting configuration of the Op-Amp in figure 4.3. Because of Vi = 0, we will have

$$V_s - V^- = V_s - RI = 0.$$

Considering that the output voltage V_o is

$$V_o = (R_f + R)I,$$

we can use the expression of I to obtain

$$V_s = \frac{R}{R+R_f}V_o$$

The output voltage V_o and voltage gain A will be

$$V_o = AV_i, \qquad A = 1 + \frac{R_f}{R}.$$

Considering that in this configuration V_s is directly connected to V^+ and V^- is not a virtual ground, the input impedance of the amplifier is $R_i + R$, where R_i is the real input impedance of the Op-Amp.

4.3.2 Inverting Amplifier

This circuit has been already discussed in section 4.2.3. For completeness, the solution and some comments are here reported

$$V_o = AV_i, \qquad A = -\frac{R_f}{R}$$

It is worthwhile to notice that because $V- = 0$, the circuit input impedance is just R. Having values of R typically of few kΩ, the inverting configuration doesn't preserve the high impedance characteristic of an Op-Amp. A connection of the circuit input to a network can potentially create appreciable perturbations.

4.3.2.1 Inverting Summing Stage

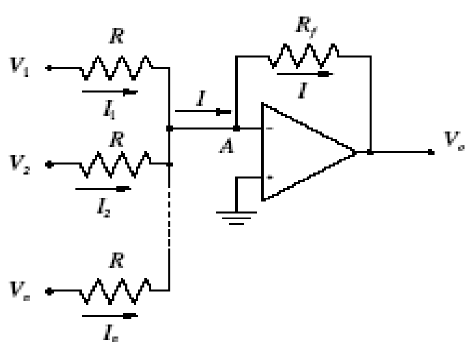

Figure 4.4: Summing stage using an Op-Amp.

Figure 5.1 shows the typical configuration of an inverting summing stage using an Op-Amp. Using the virtual ground rule for node A and Ohm's law we have

$$I_n = \frac{V_n}{R} \qquad , \qquad I = \sum_{n=1}^{N} I_n$$

Considering that the output V_o is

$$V_o = -R_f I,$$

we will have

$$V_o = A\sum_{n=1}^{N} V_n \qquad A = -\frac{R_f}{R}$$

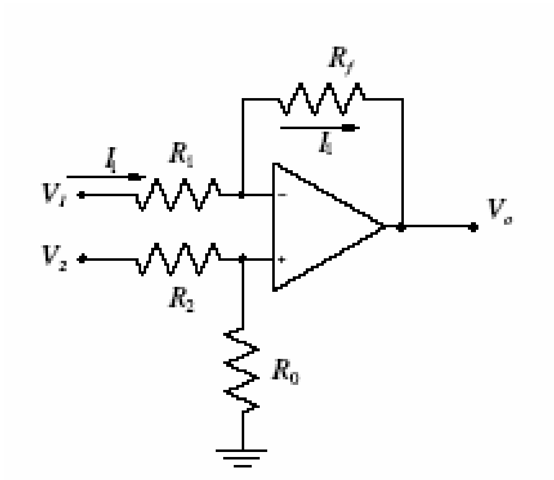

Figure 4.5: Differential input configuration of the Op-Amp

4.3.3 Differential Input Stage

Let's now solve the differential input circuit of the Op-Amp in figure 4.5. Writing the voltage drop across R_1 and R_f, we obtain the linear system

$$V^- - V_1 = R_1 I$$
$$V_o - V^+ = R_f I$$

Solving the system with respect to V_o, we get

$$V_o = \left(1 + \frac{R_f}{R_1}\right)V^- - \frac{R_f}{R_1}V_1$$

Using the voltage divider equation to obtain V^+ and because $V^+ - V^- = 0$, we have

$$V^- = V^+ = \frac{R_o}{R_2 + R_o} V_2$$

And finally, we get

$$V_o = \frac{R_1 + R_f}{R_1} \frac{R_o}{R_2 + R_o} V_2 - \frac{R_f}{R_1} V_1$$

A way to to obtain the same voltage gain for V_2 and V_1 is to impose $R_0 = R_f$ and $R_1 = R_2 = R$. The output voltage becomes

$$V_o = A(V_2 - V_1) \qquad A = \frac{R_f}{R}$$

This differential configuration is not very convenient because it does not preserve the high input impedance of the Op-Amp. In fact , considering that the Op-Amp input impedance is very high, we have that the resistance seen from V_1 is $R_2 + R_0$. Usually, the sum of those resistors is at least one order of magnitude smaller than the Op-Amp input impedance. Moreover, if we need to build a variable gain differential amplifier, we will need to change more than one resistor value. Matching the resistances values can become an issue when thermal drifts become important. More practical and stable configurations called Instrumentation amplifiers are available "off the shelf".

4.3.4 Voltage Follower (Unity Gain "Buffer")

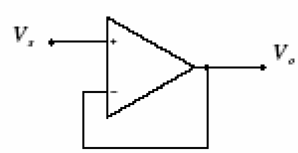

Figure 4.6: Voltage Follower or unity gain buffer

The circuit sketched in figure 4.6 is called voltage follower or unity gain buffer. The feedback line with no load gives

$$V_o = V^+.$$

Moreover, because of the condition $V_i = 0$ we will have

$$V^- = V^+,$$

which implies

$$V_o = V_i.$$

The output follows the input voltage with unitary gain.

Considering that the high impedance input and the low impedance output values of Op-Amps are close to the state of the art in the electronic design, the voltage follower can be used as an isolation stage (buffer) between two circuits.

4.3.2.3 Basic Instrumentation Amplifier

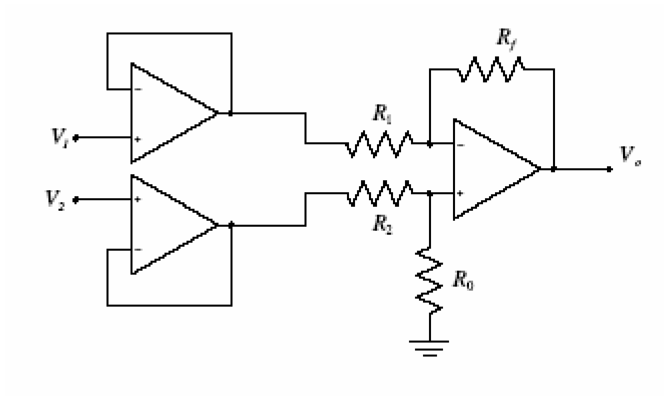

Figure 4.7:Basic Instrumentation amplifier circuit

Instrumentation amplifiers are designed to have the following characteristics: differential input, very high input impedance, very low output impedance, variable gain, and good thermal stability. Because of those characteristics they are suitable to be used as input stages of electronics instruments.

Figure 4.7 shows a configuration of three operational amplifier necessary to build a basic instrumentation amplifier.

5.1.3 Voltage to Current Converter (Transconductance Amplifier)

A voltage to current converter is an amplifier that produces a current proportional to the input voltage. The constant of proportionality is usually called *transconductance.* Figure 4.8 shows a Transconductance Op-Amp, which is nothing but a non inverting Op-Amp scheme.

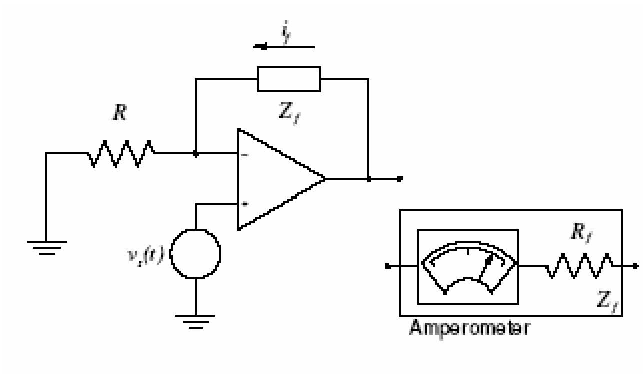

Figure 4.8: Basic Transconductance amplifier circuit

The current flowing through the impedance Z_f is proportional to the voltage V_s. In fact, supposing the infinite input impedance of the Op-Amp, we will have

$$i_f(t) = \frac{V_s(t)}{R_f}.$$

Placing an amperometer in series with a resistor with large resistance as a feedback impedance, we will have a high resistance voltmeter. In other words, the induced perturbation of such circuit will be very small because of the very high impedance of the operational amplifier.

5.1.4 Current to Voltage Converter (Transresistance Amplifier)

A current to voltage converter is an amplifier that produces a voltage proportional to the input current. The constant of proportionality is called *transimpedance* or *transresistance,* whose units are Ω. Figure 4.9 show a basic configuration for a transimpedance Op-Amp. Due to the virtual ground the current through the shunt resistance is zero, thus the output voltage is the voltage difference across the feedback resistor R_f, i.e.

$$v_o(t) = -R_f i_s(t)$$

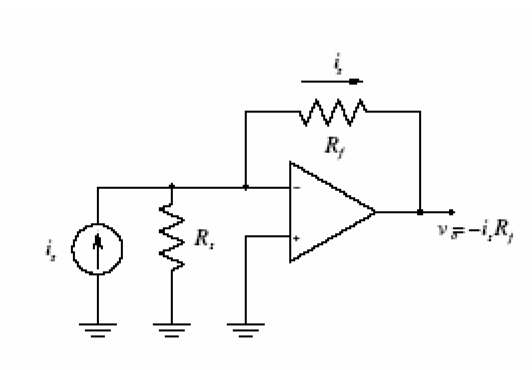

Figure 4.9: Basic transimpedance Op-Amp

Photo-multipliers photo-tubes and photodiodes drivers are a typical application for transresistance Op-amps. In fact, quite often the photocurrent produced by those devices need to be amplified and converted into a voltage before being further manipulated.

4.3.5 Integrator Amplifier

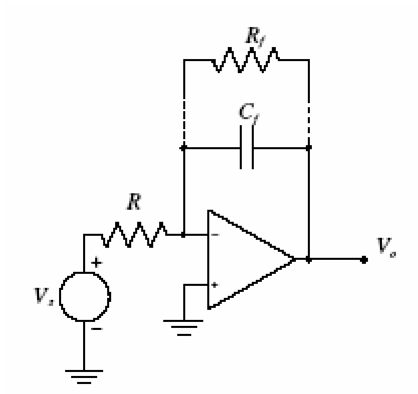

Figure 4.10: Active integration stage using an Op-Amp.

Let's consider the circuit in figure 4.10 without the resistance R_f. The voltage drop v_o across the capacitor C_f is

$$v_o(t) = -\frac{1}{C_f}\int_{-\infty}^{t} i(\tau)d\tau \qquad 4.3$$

and the current flowing through the resistance R is

$$i(t) = \frac{v_i(t)}{R}$$

Placing the expression of i(t) obtained from the previous equation into eq.(4.3), we will obtain

$$v_o(t) = -\frac{1}{\tau}\int_{-\infty}^{t} v_{in}(t')dt' \qquad \tau = R_f C$$

Real Op-Amps or signals connected to the input have often (always) a DC offset. This offset is indeed integrated and after a given time will saturate the amplifier output. This saturation is essentially a manifestation of the instability of the circuit at low frequency. Moreover, the initial charge of the capacitor is undefined, making the initial output state unpredictable.

A common way to avoid these problems is to introduce the resistance R_f in parallel with the capacitor C_f which reduces the amplifier DC gain. An intuitive way to understand the effect of this feedback resistance is that it does not allow the capacitor to be charged ad libitum. The choice of the R_f is not so trivial if we want to preserve the characteristic of good integrator.

If the DC current must be integrated, we can place a switch in parallel with the capacitor to be opened when the integration is started. In this way we will have the capacitor state completely defined.

4.3.6 Differentiator Amplifier

Let's now consider the circuit in figure 4.11 without the feedback capacitor C_f. Applying a similar analysis to that used in the integrator amplifier we will have

$$i(t) = C\frac{dv_i}{dt}$$

$$vo(t) = R_f i$$

and indeed

$$v_o(t) = \tau \frac{dv_i}{dt}, \quad \tau = R_f C$$

This configuration without C_f doesn't work well with real Op-Amps, because of stability problems. In fact, the introduction of the capacitor compromises the internal compensation of the Op-Amp. Placing a capacitor C_f in the feedback network restores the compensation making the overall circuit stable. The choice of C_f is not trivial if we want to preserve the circuit differentiator characteristics.

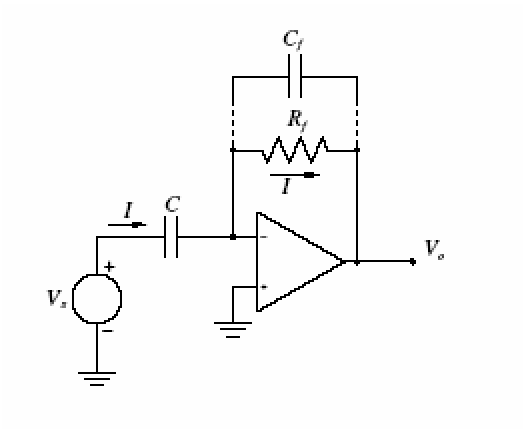

Figure 4.11: Differentiator stage using an Op-Amp.

4.3.7 Logarithmic Circuits

Combining logarithmic circuits such as logarithmic and anti-logarithmic amplifiers we can implement analog multipliers and dividers. Let's see in more details how those circuit work.

4.3.7.1 Logarithmic Amplifier

Figure 4.12 shows an elementary logarithmic amplifier, i.e. the output is proportional to the logarithm of the input. A BJT as feedback provides a larger input dynamic range. Let's analyze the logarithmic amplifier in more detail

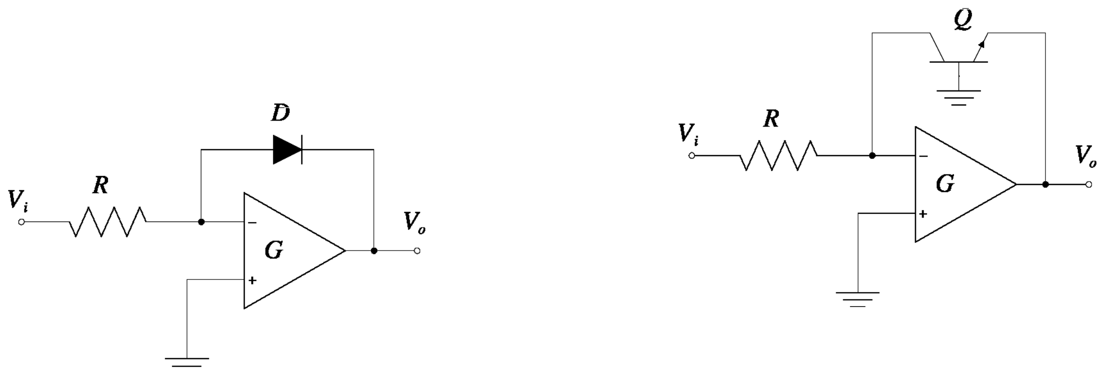

Figure 4.12: Elementary logarithmic amplifier

Because the Op-Amp is mounted as an inverting amplifier, if v_i is positive, then v_o must be negative and the diode is in conduction. We must have

$$i \cong I_s e^{-qV_o / k_B T}, \quad I_s \ll 1$$

where $q < 0$ is the electron charge. Considering that

$$i = \frac{v_i}{R},$$

and after some algebra we finally get

$$v_o = \frac{kBT}{-q}\left[\ln(v_i) - \ln(RI_s)\right]$$

The constant term $\ln(RI_s)$ is a systematic error that can estimated and subtracted at the output. It is worth to notice that v_i must be positive to have the circuit working.

4.3.7.2 Anti-Logarithmic Amplifier

Figure 4.13 shows an elementary logarithmic amplifier, i.e. the output is proportional to the inverse of logarithm of the input. Same remarks of the logarithmic amplifier about the npn BJT applies to this circuit.

The current flowing through the diode or the BJT is:

$$i \cong I_s e^{-qV_i / kBT}$$

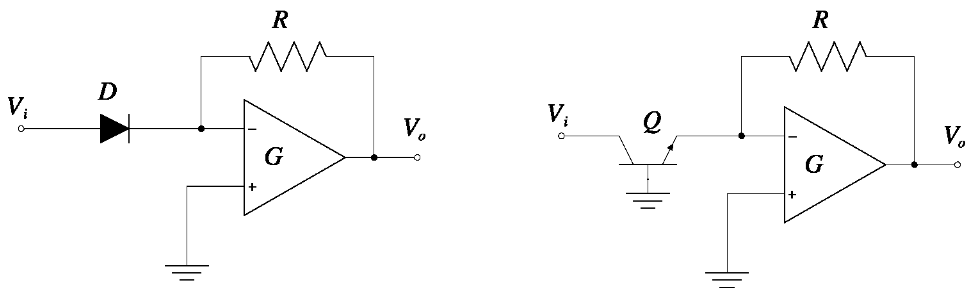

Figure 4.13: Elementary anti-logarithmic amplifier

where in the argument of the exponential function we have the input voltage. Considering that

$$v_o = -Ri$$

thus

$$v_o \cong -RI_s e^{-qV_i/kBT}.$$

If the input v_i is negative, we have to reverse the diode's connection or replace the BJT with a pnp BJT.

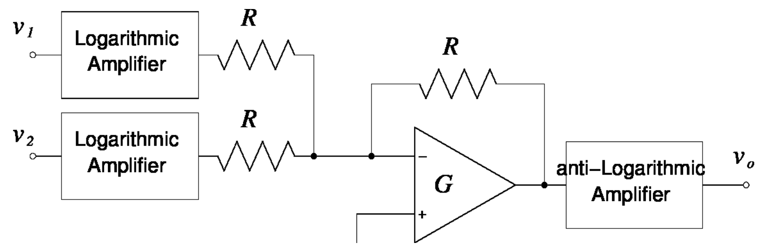

Figure 4.14: Elementary analog multiplier.

4.3.7.3 Analog Multiplier

Figure 4.14 shows an elementary analog multiplier based on a two log one anti-log and one adder circuits.

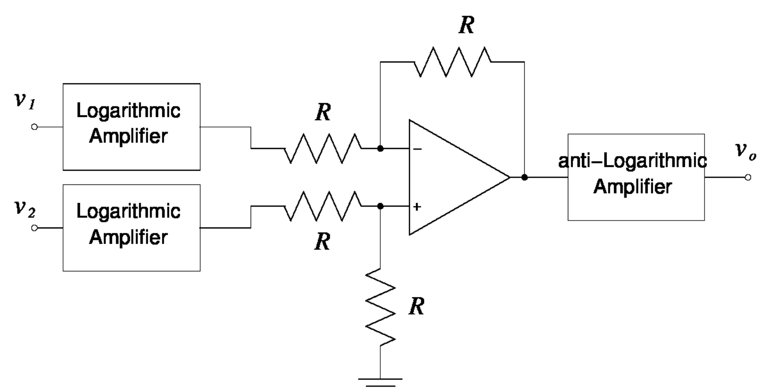

Figure 4.15: Elementary analog divider.

4.3.7.4 Analog Divider

Figure 4.15 shows an elementary logarithmic amplifier based on a two log one anti-log and one adder circuits.

4.3.8 Multiple-Feedback Band-Pass Filter

Figure 4.16 shows the so called multiple-feedback bandpass, a quite good scheme for large passband filters, i.e. moderate quality factors around 10.

Here is the recipe to get it working. Select the following parameter which define the filter characteristics, i.e the center angular frequency ω the quality factor Q or the the passband interval (ω1, ω2) , and the passband gain A_{pb}

Set the same value C for the two capacitors and compute the resistance values

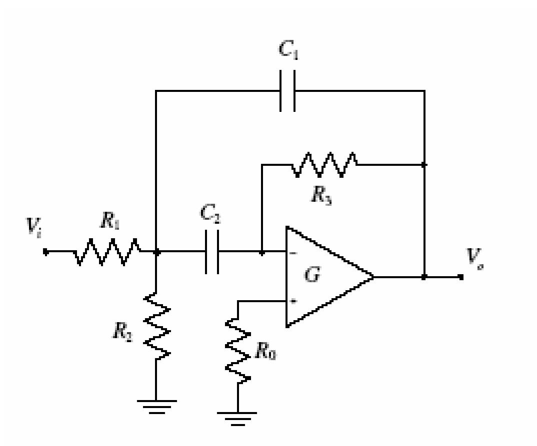

Figure 4.16: Multiple-feedback band-pass filter

$$R_1 = \frac{Q}{\omega_o C A_{pb}}$$

$$R_2 = \frac{Q}{\omega_o C(2Q^2 - A_{pb})}$$

$$R_3 = \frac{Q}{\omega_o C}$$

Verify that

$$Ap_b = 2\frac{R_3}{R_1} < 2Q^2$$

4.3.9 Peak and Peak-to-Peak Detectors

The peak detector circuit is shown in figure 4.17. The basic ideal is to implement an integrator circuit with a memory.

The Op-amp A_0 is essentially a unity gain voltage follower that charges the capacitor C up to the peak voltage.

The diode D_0 and the A_1 Op-Amp high input impedance prevent the fast discharge of the capacitor.

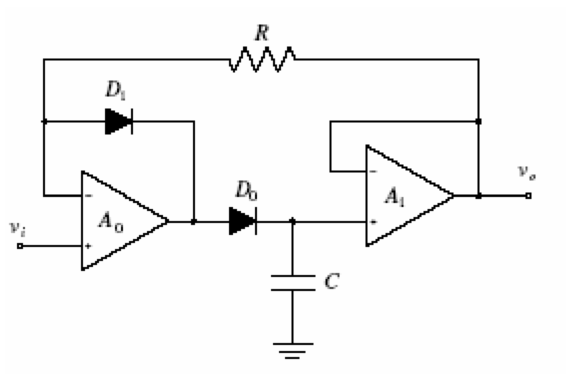

Figure 4.17 Peak Detector Circuit

The resistance R provides a feedback path to the Op-Amp A_1 when its output is less than the peak voltage, avoiding saturation.

The Op-Amp A_1 is a unity gain follower that acts as a buffer between the capacitor C and the output, preventing the discharge of C through load resistance connected to the output of A_1.

The Op-Amp A_0 should have a high slew rate (20 V/μs) to avoid the maximum voltage being limited by the slew rate.

It is worthwhile to notice that if D_0 and D_1 are reversed the circuit becomes a negative peak detector.

Using a positive and a negative peak detector as the input of a differential amplifier stage we can build a peak-to-peak detector.

4.3.10 Zero Crossing Detector

When vi is positive and because it is connected to the negative input then v_o becomes negative and the diode D1 is forward biased and conducting..

4.3.11 Analog Comparator

An *analog comparator* or simply *comparator* is a circuit with two inputs vi , vref and one output $v_o$which fulfills the following characteristic:

An Op-Amp with no feedback behaves like a comparator. In fact, if we apply a voltage $v_i > v_{ref}$, then $V^+ - V^- = v_i - v_{ref} > 0$. Because of the high gain, the Op-Amp will set v_o to its maximum value $+V_{sat}$ which is a value close to the positive voltage of the power supply. If $v_i < v_{ref}$, then $v_o = -v_{sat.}$ The magnitude of the saturation voltage are typically about 1V less than the supplies voltages.
Depending on which input we use as voltage reference $v_{ref,}$ the Op amp can be an inverting or a non inverting analog comparator.

4.3.12 Regenerative Comparator (The Schmitt Trigger)

The *Regenerative comparator* or *Schmitt Trigger* shown in figure 4.18 is a comparator circuit with hysteresis.
It is worthwhile to notice that the circuit has a positive feedback. With positive feedback, the gain becomes larger than the open loop gain making the comparator swinging faster to one of the saturation levels.
Considering the current flowing through R_1 and R_2,we have

$$I = \frac{V_1 - V^+}{R_2} = \frac{V^+ - V_0}{R_1} \Rightarrow V^+ = \frac{V_1 R_1 + V_o R_2}{R_1 + R_2}$$

The output V_o can have two values, $\pm V_{sat.}$ Consequently, V^+ will assume just two trip point values

$$V^{+(utp)} = \frac{V_1 R_1 + V_{sat} R_2}{R_1 + R_2} \qquad V^{+(upt)} = \frac{V_1 R_1 - V_{sat} R_2}{R_1 + R_2}$$

When $V_i < V^{+(utp)}$, V_o is high, and when $Vi < V^{+(ltp)}$, V_o is low.

To set $V^+ = 0$ it requires that

$$V_1 = -\frac{R_2}{R_1} V_o$$

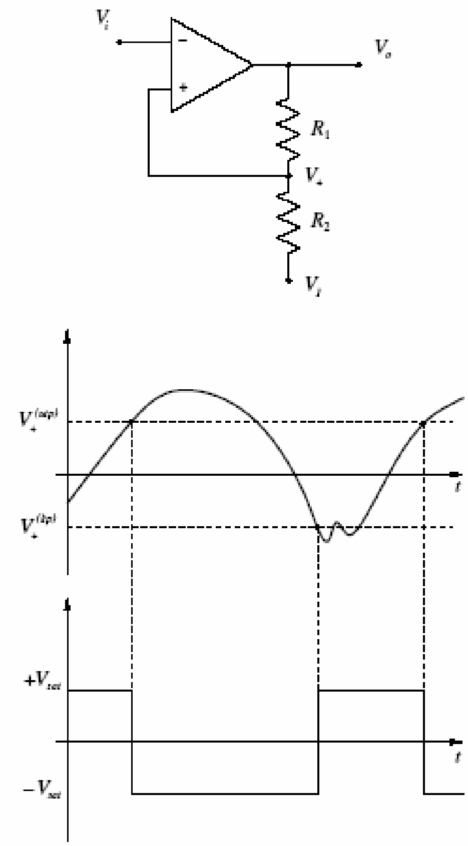

Figure 4.18: Schmitt Trigger

This circuit is usually used to drive an analog to digital converter (ADC). In fact, jittering of the input signal due to noise which prevents from keeping the output constant, will be eliminated by the hysteresis of the Schmitt trigger (values between the trip points will not affect the output).

Example1: ($V_{sat} = 15V$)

Supposing we want to have the trip points to be $V^+ = \pm 1.5V$, if we set $V_1 = 0$ then $R_2 = 9R_1$

4.4 The Real Op-Amp

Lets consider in this section a more realistic model of the Op-Amp by including a finite input impedance R_i, non zero output impedance R_o, finite gain A, bias currents and voltage offsets. Using ideal components, the equivalent circuit of the real Op-Amp is shown in figure 4.19.

4.4.1 Bias Currents and Voltage and Current Offsets

Imbalances inside of the Op-Amp, mainly due to differences in the electronics components, produce undesirable bias currents and a voltage offset at the inputs. Input voltage and current offsets can be modeled by introducing ideal generators as shown in figure 4.19. Current Offset is defined as the difference in the magnitude of the bias currents, i.e.

$$ios = \left|ib^{+}\right| - \left|ib^{-}\right|$$

A way to characterize the voltage offset is to use the voltage follower configuration with the input V_i connected to the ground. The voltage offset will be directly the output voltage V_o.

Current input biases can be studied connecting a resistor between the one input and ground and measuring the voltage drop across the resistor.

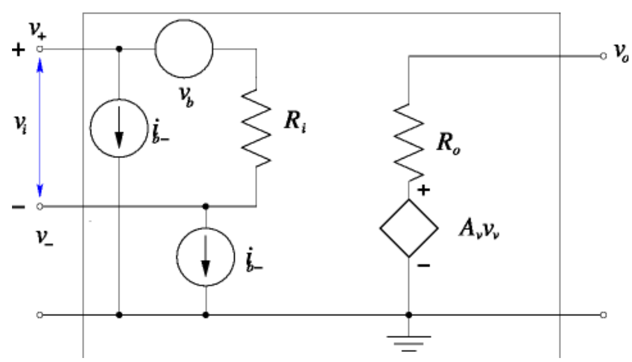

Figure 4.19: Equivalent circuit for an Op-Amp using ideal components. Voltage offsets and current biases are taken into account using ideal voltage and current generators.

4.4.2 The Common Mode Rejection Ratio (CMRR)

We want characterize the rejection of an Op-Amp output as a differential amplifier, of signals sent to both inputs. For an ideal Op-Amp we expect to obtain $V_o = 0$ for all frequencies, i.e. a perfect rejection. To define a convenient parameter which measures the rejection it is necessary to define the following ones, the *common mode gain*

$$A_C(\omega) = \frac{V_o}{V^{+} - V^{-}}, \quad V^{+} = V^{-} = V_s \sin(\omega t)$$

and the *differential mode gain*

$$A_D(\omega) = \frac{V_o}{V^+ - V^-}, \qquad V^+ = V_s \, sin(\omega t), \; V^- = 0$$

The *Common Mode Rejection Ratio* (CMRR) is defined as the modulus of the ratio of the differential gain A_D over the common mode gain A_C, i.e.

$$CMRR(\omega) = \left| \frac{A_D(\omega)}{A_C(\omega)} \right|$$

Ideally, the CMRR should be infinity for all frequencies.
This parameter can be measured using the Op-Amp differential configuration (see figure 4.5) and measuring A_C and A_D as a function of the frequency. To minimize possible large systematic errors, it is necessary to have the same gain for the two inputs V_1 and V_2. This can be achieved by placing a trimmer in the voltage divider mesh of the differential configuration circuit. Adjusting the trimmer we can minimize V_o for a single frequency and study the CMRR for a given bandwidth.
Figure 4.20 shows the CMRR as a function of frequency of a typical Op-Amp. A typical value for CMRR is 90 dB.

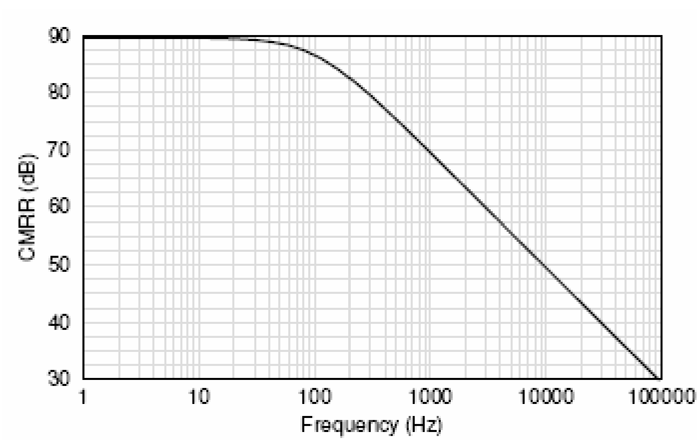

Figure 4.20: CMRR as a function of frequency of a typical Op-Amp

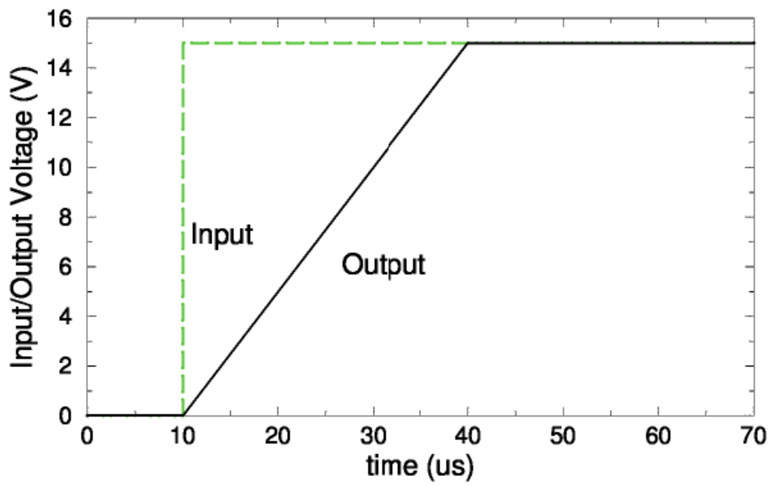

Figure 4.21: Slew rate illustration. Voltage Response v_o of the Op-Amp to a large step input voltage v_i.

4.4.5 The Slew Rate (SR)

The slew rate is defined as the maximum rate of the output voltage v_o per unit time

$$SR = \max\left\{\frac{\Delta v_0(v_i)}{\Delta t}\right\}$$

This parameter essentially measures the ability of an Op-Amp to follow voltage changes for large voltage inputs.
The slew rate can be easily observed sending a square wave to the Op-Amp input v_i, and looking at the raising and falling slope of the output signal v_o. If the slopes do not change changing the input amplitude, then the Op-Amp is slew rate limited.
A similar procedure can be applied using a low frequency sinusoidal signal as input. In this case if we increase to much the input amplitude, the output will become distorted.
The slew rate is a non-linear effect intrinsic of the architecture of the Op-Amp. In a compensated Op-Amp it is often proportional to the capacitance of the compensating network

of the gain stage. Typical slew rate values are of the order of few V/µs.

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of the gain stage. Typical slew rate values are of the order of few V/µs.

4.4.6 Ideal versus Real and Practical Considerations

The following table summarizes the main characteristic of an ideal Op-Amp together with those of typical real Op-Amp. In some cases we can found Op-Amps excelling some of the mentioned characteristics, and often being detrimental to others.

Property	Ideal Op-Amp	Typical Op-Amp
Open-loop DC Gain, A_o	∞	$>10^4$
Open-Loop Bandwidth	∞	~ 10Hz
Common Mode Rejection Ratio, CMMR	∞	> 70dB
Input Resistance R_i	∞	> 10MΩ
Output Resistance, R_o	0	<500 Ω
Input Current, $\delta I_\pm$	0	< 0.5µA
Input Offset Voltage, $\delta V_\pm$	0	< 10mV
Input Offset Current, $\delta I_\pm$	0	< 200pA

What are the conditions that dictate the range of the feedback impedances R_f? Apart from special cases, the feedback current I should be only a small fraction of the maximum output current I_o, i.e. $I = 1\% I_o$. A typical Op-Amp has a maximum output of 10mA at 10V, i.e.

$$Rf = \frac{V}{I} = \frac{10V}{10 * 0.1\% mA} = 100k\Omega$$

Typical feedback resistors should be in the range of R_f = 50 - 1MΩ.

Small difference on the differential stage of the Op-Amp produces a DC offset δV at the input, which can produce large DC output if the gain is extremely high. For example, if we have

$$\delta V = 10mV, \quad G > 10^4 \Rightarrow \quad Vo \geq 10V$$