The Geometric Viewpoint

A Survey of Geometries

Thomas Q. Sibley

Addison-Wesley Higher Mathematics

Sibley The Geometric Viewpoint *A Survey of Geometries*

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Preface

I begin to understand that while logic is a most excellent guide in governing our reason, it does not, as regards stimulation to discovery, compare with the power of sharp distinction which belongs to geometry. —*Galileo Galilei*

Geometry combines visual delights and powerful abstractions, concrete intuitions and general theories, historical perspective and contemporary applications, and surprising insights and satisfying certainty. In this textbook, I try to weave together these facets of geometry. I also want to convey the multiple connections that different topics in geometry have with each other and that geometry has with other areas of mathematics. These connections link different chapters together without sacrificing the survey nature of the whole text.

The enduring appeal and importance of geometry stem from its synthesis of intuition and reasoning. Mathematical intuition is as hard for mathematics majors to develop as is the art of proving theorems. Geometry is an ideal subject for developing both. However, geometry courses and texts for mathematics majors usually emphasize reasoning. This book strives to build students' intuition and reasoning through the text and the variety of problems and projects. The dynamic geometry software now available provides one valuable way for students to build their intuition and prepare them for proofs. Thus problems dependent on technology join hands-on explorations, proofs, and other problems. I see this book as building on the momentum of the NCTM *Standards* for primary and secondary education and on the calculus reform movement in its goal of helping students internalize mathematical concepts and thinking.

- Proofs end with the symbol ■. Examples end with the symbol ●. We abbreviate "if and only if" with "iff." In a definition, the word being defined is italicized.

Prerequisites.

The major prerequisite for using this text is the ability to read proofs and other mathematical content at the level of an undergraduate mathematics major. The book assumes a familiarity with high school algebra and geometry, although a detailed memory of geometry theorems isn't expected. (Appendix A supplies many of them in Euclid's language and in sometimes more modern phrasing.) Calculus is needed for Section 2.4 and, as noted, occasionally in examples, problems, and projects. An understanding of vectors is needed for Sections 2.3 and 2.5. Linear algebra is needed for Sections 4.3, 4.4, 4.5, 6.3, 6.4, 6.5, 6.6, and 7.4. We discuss groups in Chapters 4, 5, and 6, and fields in Section 7.4, but students aren't expected to be familiar with them.

To the Student

Everyday speech equates intuitive with easy and obvious. However, recent psychology research confirms what mathematicians have always understood: People build their own intuitions through reflection on new experiences. My students often describe the process as learning to think in a new geometry. I hope the text's explanations are clear and provide new insights. I also hope you find many nonroutine problems you can solve after an extended effort; such challenges enrich your intuition.

Problems.

Learning mathematics centers on doing mathematics, so problems are the heart of any mathematics textbook. I hope that you (and your teachers) will enjoy spending time pondering and discussing the problems. Make lots of diagrams and, when relevant, physical or computer models. The problems are varied and include some that encourage hands-on experimentation and conjecturing, as well as more traditional proofs and computations. Answers to selected problems appear in the back of the book.

Projects.

Essays, paper topics, and more extended and open-ended problems appear at the end of each chapter as projects. Too often textbooks shift to a new topic just when students are ready to make their own connections. These projects encourage extensions of the ideas discussed in the text. Many of these projects benefit from group efforts.

Visualization.

This text has an unusually high number of figures because I believe that students benefit greatly from visual thinking. In addition, you will be asked to draw your own designs, to use a computer, or to build three-dimensional models. Visualization not only builds your intuition about the topic at hand, it can also lead you to new insights.

History.

Geometry reveals the rich influences over the centuries between mathematics and other fields. Students in geometry, even more than other areas of mathematics, benefit from historical background, which I have included in the first section of each chapter and the biographies. The biographies give a personal flavor to some of the work discussed in the text. One common thread I noted in reading about these geometers was the importance of intuition and visual thinking. As a student, I sometimes questioned my mathematical ability because I needed to visualize and to construct my own intuitive understanding instead of grasping abstract ideas directly. Now I realize that great mathematicians also built on intuition and visualization for their abstract proofs and theories. Perhaps this understanding will help you as well.

Readings and Media.

Geometry is a field blessed with many wonderful and accessible expository writings and a long tradition of visual materials. I hope that you find the list of readings and media at the end of each chapter valuable while you are studying from this book and later as a resource.

To the Teacher

Suggested Course Outlines.

This survey text supports various courses. If time permits, one option is to do the entire book in sequence. Even an entire year devoted to this course would require that you pick carefully among the problems and projects in order to cover all the chapters. One-quarter courses could cover two chapters—for example, Chapters 1 and 3, Chapters 4 and 5, or shortened versions of the following semester course options.

- Course 1, teacher preparation: Chapters 1, 3, and 4, and, as needed, Sections 2.1 and 2.2. Sections 3.5 and 4.6 are optional. If time permits, topics from Chapters 2, 5, and 7 may be included.
- Course 2, Euclidean geometry: Chapters 1, 2, 4 (except Section 4.6), and as much of 5 as time permits.
- Course 3, transformational geometry: Chapters 4, 5, and 6, and, as needed, Sections 1.1, 1.2, 1.5, 1.6, 2.1, 2.2, and 2.5.
- Course 4, axiomatic systems and models: Chapters 1 and 3, Sections 2.1, 6.1, 6.2, 6.3, 7.1, 7.2, 7.4, and parts of Sections 2.3 and 6.5.

More than one class period should be devoted to most of the sections. In the first period I usually discuss the main points of the section, model one or more of the problems or relevant projects, and/or have the students do one of the group projects. I often use part of the next class (or classes) for group presentations of problems or projects, which can spark discussion about different approaches. I hold class in our computer lab several times during the semester. I encourage students to use their graphing calculators for matrix computations in Chapters 4 and 6.

The projects serve several pedagogical purposes. Some of them are intended as group explorations to introduce topics in class. Others are suitable for essay topics or classroom discussion. Others are open-ended or extended problems. The most succinct ones, of the form "Investigate . . . ," are leads for papers. I have assigned papers in my geometry class for more than 10 years. Students uniformly regard these papers as the high point of the class.

The chapters are largely independent, although they are often interconnected. Chapter 3 depends on the first four sections of Chapter 1. Section 4.6 relies on Section 3.1.

Chapter 5 requires the first two sections of Chapter 4. Sections 6.3–6.6 depend on Chapter 4, except Section 4.6. Sections 6.5 and 6.6 also rely on Section 3.1. Section 7.4 requires Chapters 4 and 6.

Instructor's Resource Manual.
Because of the unusual variety of problems and projects, answers to the problems and selected projects and suggestions for their use can be found in the *Instructor's Resource Manual,* available from Addison Wesley Longman. Instructor support for integrating Geometer's Sketchpad into the geometry course will be included.

ACKNOWLEDGMENTS

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I particularly want to thank three people. Paul Krueger not only painstakingly made many of the fine illustrations that are an integral part of this book, but he also pondered with me over the years the connections of geometry with other fields. Connie Gerads Fournelle helped greatly with her valuable perspective on the text, both as a student using an early version and later as my assistant writing answers to selected problems. My wife, Jennifer Galovich, provided the unswerving support and love I needed to keep chipping away at this project, even when no end to it seemed discernible.

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