

$(\frac{1}{4})^{1-d} = c(\frac{1}{3})^{1-d}(\frac{1}{4})^{1-d}$. Substituting the first equation into the second yields $(\frac{1}{4})^{1-d} = \frac{1}{3} \cdot (3)^d$ or $4 = 3^d$. Then $d = \log 4 / \log 3$, as before. ●

The exponent $1 - d$ in Richardson's equation isn't as mysterious as it may seem. If you divide a unit f , the estimated length $L(f)$ of a self-similar curve should be the number of segments times their length. Moreover, the number of units is proportional to $n = (\frac{1}{f})^d$, that is, $L(f) = c \cdot n \cdot f = c \cdot (1/f)^d \cdot f = c \cdot f^{1-d}$. We want to solve Richardson's equation for d , based on empirical values for f and $L(f)$. With two unknowns, c and d , we need two values of f and $L(f)$. From $L(f_1) = c(f_1)^{1-d}$ and $L(f_2) = c(f_2)^{1-d}$, $L(f_1)/L(f_2) = (f_1/f_2)^{1-d}$ and finally

$$d = 1 - \frac{\log[L(f_1)/L(f_2)]}{\log(f_1/f_2)}. \quad (5.4)$$

Figure 5.50 shows the coastline of England and Wales from Bristol to Liverpool. If we estimate the length of this coastline by using $f_1 = 57$ -mi segments, we need 5 segments, so $L(f_1) \approx 285$ mi. For $f_2 = 28.5$ mi, we need $12\frac{1}{2}$ segments, so $L(f_2) \approx 356$ mi; $f_3 = 14.25$ mi gives 32 segments and $L(f_3) \approx 456$ mi. As we shorten the segment length, we can follow the contour better, taking into account more and more of the multitude of peninsulas and bays. When we use f_1 and f_2 , Eq. 5.4 gives $d \approx \log(285/356) / \log(57/28.5) \approx 1.32$. ●

That $d \approx 1.36$ when you use f_2 and f_3 and $d \approx 1.34$ when you use f_1 and f_3 .

The values of d that Richardson found for coastlines are remarkably stable over a wide range of scales f . (Such values can't be stable over all possible scales unless the object is truly self-similar.)

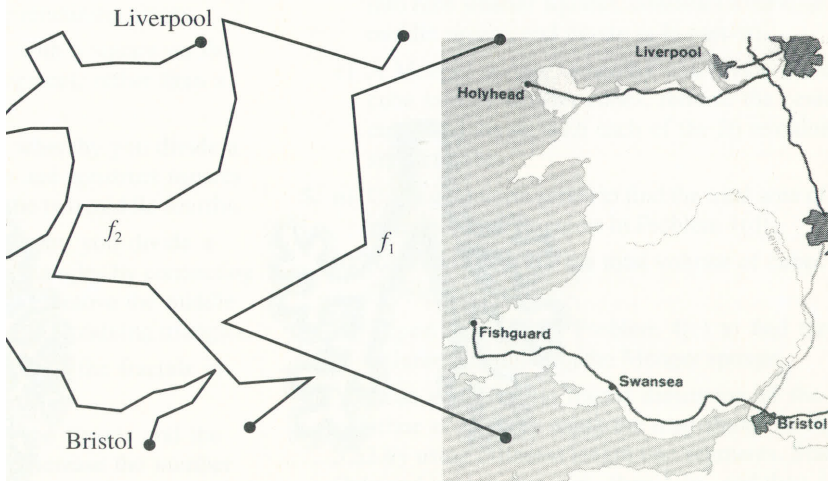


Figure 5.50 The mapped coastline of Wales and part of England and approximate outlines produced by using segments of different lengths.