

Index

Notation

$\widehat{AB}$ Arc of a circle from A to B , 49
 $\overline{AB}$ Length of segment $\overline{AB}$, 3
 $\overleftrightarrow{AB}$ Line on A and B , 3
 $\overline{AB}$ Line segment between A and B , 3, 293
 $\overrightarrow{AB}$ Ray from A through B , 25, 294
 $\angle ABC$ Angle ABC with vertex at B , 3, 294
 $\triangle ABC$ Triangle with vertices A , B , and C , 3, 294
 $AB//CD$ A and B separate C and D , 232
 $\mathbf{AF}^2$ Affine plane over the field $\mathbf{F}$, 279
 α^{-1} The inverse of α , 135
 $\mathbf{C}$ The complex numbers, 65
 $\mathbf{C}^\#$ The extended complex numbers, 170
 $\mathbf{C}_n$ The cyclic group with n elements, 185
 $d(A, B)$ Distance from A to B , 62
 d_H Hyperbolic distance, 121, 250
 d_T Taxicab distance, 31, 78
 $\mathbf{D}_n$ The dihedral group with n rotations, 185
 $\overline{\mathbf{D}}'_n$ The symmetry group of a rectangular prism, 203
 $\mathbf{F}$ A field, 279
 $\langle g_1, g_2, \dots, g_n \rangle$ Generators of a group, 191
 $H(PQ, RS)$ Harmonic set, 228
 $k \cdot l$ The point of intersection of lines k and l , 231
 $m\angle ABC$ Measure of angle $\angle ABC$, 14
 $n!$ n factorial, product of first n positive integers, 205

$\overline{\mathbf{P}}$ The icosahedral group, 203
 $\tilde{P}$ Oriented point, 250
 $\mathbf{PF}^2$ The projective plane over the field $\mathbf{F}$, 281
 $\mathbf{Q}$ The rational numbers, 76
 $\mathbf{R}$ The real numbers, 62
 $\mathbf{S}_n$ The symmetric group on n elements, 205
 $\overline{\mathbf{T}}$ The tetrahedral group, 203
 $\vec{v}$ Vector, 85
 $\overline{\mathbf{W}}$ The octahedral group, 203
 $\|(x, y)\|$ Length of vector (x, y) , 85
 $\mathbf{Z}$ The integers, 77
 $\mathbf{Z}_n$ The integers modulo n , 279
 $\cong$ Congruent, 3
 $\parallel$ Parallel, 12

Terms

AAS, 12
 Euclid I–26, 290
 Absolute conic, 249
 Achilles and the Tortoise, 4
 Affine matrix, 147, 164
 Affine plane, 252, 267, 279
 Affine transformation, 147, 252, 282
 Analytic,
 geometry, 59ff
 Greek meaning, 2
 model, 61ff, 238ff, 279ff
 Angle of parallelism, 108

- Angle sum, 3
- Antiprism, 206
- Archimedean solids, 55
- Area axioms, 117
- Asymptote, 71
- Attracting fixed point, 136
- Axiomatic system, 24
- Axioms, 25
- Babylonian mathematics, 2
- Balanced incomplete block design (BIBD), 272
- Barycentric coordinates, 74
- Base, 40
- Between, betweenness, 23, 35, 257, 293
- BIBD, 272
- Bilateral symmetry, 180
- Block, 272
- CAD, 78, 165, 254
- Cavalieri's Principle, 50
- Center of inversion, 167
- Central,
 - angle, 19
 - symmetry, 152, 166
- Characteristic axiom of hyperbolic geometry, 99
- Chemistry, 206ff
- Circular points at infinity, 252
- Circumscribed, 18
- Closure, 136
- Code, 276
- Code word, 276
- Collinear, 231
- Collineation, 244, 254, 281
- Color symmetry, 189, 199
- Color-preserving symmetry, 198
- Color-switching symmetry, 199
- Combinatorics, 266
- Complete, 31
 - quadrangle, 228
 - quadrilateral, 229
- Complex numbers, conjugates, modulus, 65
- Composition, 135
- Computer-aided design (CAD), 78, 165, 254ff
- Concurrent, 231
- Congruence, 12, 111
- Conic, 65, 240ff, 260
- Conjugate, 65, 152
- Consistent, 31
- Constructible, 20
- Construction, 9ff
- Contraction mapping, 158
- Converse, 13
- Convex, 26, 257
 - function, 65
- Cross polytope, 89
- Cross-ratio, 239ff
- Crystal, 181, 204, 207ff
- Crystallographic restriction, 194, 204
- Cyclic group, 185
- Decomposition, 16
- Defect, 118
- Degenerate conic, 69
- Deltahedron, 56
- Desargues's theorem, 229, 247, 260
- Descartes's formula, 42
- Design theory, 272ff
- Diagonal point, 237
- Differential geometry, 102
- Dihedral group, 185
- Dilation, dilatation, 159
- Direct isometry, 140
- Directrix, 66
- Discrete, 190
- Dodecahedron, 41
- Doubling a cube, 14
- Dual, 56, 92, 235, 271
- Duality, 235, 271
- Edge, 39
- Egyptian Mathematics, 2
- Eigenvalue, 150
- Eigenvector, 150
- Ellipse, 66
- Equivalence relation, 176
- Equivalent,
 - logically, 13
 - polygons, 117
- Erlanger Programm, 142
- Error correcting code, 276
- Euclid's *Elements*, 2, 9ff, 287ff
- Euler's formula, 42
- Excess, 48, 125
- Face, 39
- Field, 278
- Fifth postulate, 13, 288
- Fixed point, 133, 244

- Flow chart for wallpaper patterns, 196
- Foci, focus, 66
- Fractal, 156, 214ff
 - curve, 220
 - dimension, 220
 - surface, 220
- Frequency, 43
- Frieze pattern, 190
- Generator, 191
- Geodesic, 102
 - dome, 43ff
- Glide reflection, 140
- Golden ratio, 6
- Great circle, 46
- Group, 136
- Half-plane model, 105
- Hamming distance, 276
- Harmonic set, 228
- Hausdorff dimension, 216
- Hermite curve, 82
- Hilbert's axioms, 23, 293
- h -inner product, 251
- h -length, 251
- Homogeneous coordinates, 238
- h -orthogonal, 251
- Hyperbola, 66
- Hyperbolic,
 - distance, 121, 250
 - geometry, 98ff, 249
 - isometries, 250
 - planes, 284
- Hyperboloid, 86
- Hypercube, 88
- Hyperplanes, 86, 164
- Icosahedron, 41
- Ideal point, 226
- Identity, 135
- IFS, 158
- IFS fractal, 158
- Incommensurable, 3
- Independent, 31
- Indirect isometry, 140
- Inscribed, 17
- Interior, 249, 284
- Interpretation, 29
- Invariant, 133
- Inverse, 135
- Inversion, 167ff
- Inversive plane, 167
- Invertible, 147
- Irrational, 3
- Isomer, 212
- Isometry, 137ff, 161, 165
- Isomorphic, 33
- Iterated function system (IFS), 158
- Kirkman's schoolgirl problem, 265
- Klein model, 104
- Klein's definition of geometry, 142ff
- Koch curve, 214ff
- LaGrange's theorem, 186
- Law of cosines, 20
- Lightlike, 214
- Line conic, 242
- Locus, 65
- Lorentz transformations, 212, 258
- Lunes, 7, 46
- Median, 63
- Metamathematics, 30
- Metatheorem, 30
- Mickelson-Morley experiment, 211
- Midline, 191
- Minkowski geometry, 212, 258
- Mirror reflection, 138
- Möbius transformation, 171ff
- Mod, 279
- Model, 29ff
- Modulo, 279
- Motif, 181
- Napoleon's theorem, 94
- n -gon, 17
- Non-Euclidean geometry, 97ff
- Nonorientable, 127, 256
- Nonsingular; *see* invertible.
- Octahedron, 41
- Omega,
 - point, 109, 250
 - triangle, 109
- One-to-one, 133
- Onto, 133
- Orbit, 186
- Order,
 - field, 279

Order, (*cont.*)
 plane, 267, 271
 Orientable, 127
 Orientation, 132
 Oriented point, 256
 Oriented projective geometry, 256ff
 Orthogonal,
 circles, 103
 matrix, 164
 Oval, 282

 Parabola, 66
 Parallel, 12, 267, 288
 Parallelogram, 18
 Parametric equations, 72
 Pascal's theorem, 260
 Pasch's axiom, 23, 110, 293
 Penrose tiling, 209
 Pentagram, 6
 Periodic point, 136
 Perspective,
 from a line, 229
 from a point, 229
 in art, 226, 230
 in computer-aided design, 254ff
 Perspectivity, 227
 Pick's theorem, 93
 Playfair's axiom, 12, 295
 Poincaré model, 104
 Polar coordinates, 73
 Polygon, 17
 Polyhedron, 39
 Polytope, 88
 Projective plane, 238, 270, 281
 Projectivity, 236, 243
 Proof, 26ff
 Proportional, 35
 Pseudosphere, 105
 Pyramid, 39
 Pythagorean theorem, 7, 292

 Quadratic formula, 6
 Quasicrystal, 209

 Ratio of proportionality, 35
 Ray, 25, 294
 Reflection property for parabola, 69
 Regular polygon, 17
 Regular polyhedron, 41
 Relatively consistent, 31

Relativity, theory of, 210ff
 Repelling fixed point, 136
 Rotation, 138
 Rotatory reflection, 162

 Saccheri quadrilateral, 113
 SAS, 12, 295
 Euclid I-4, 289
 Scaling ratio, 153
 Screw motion, 164
 Self-similar, 215
 Semiregular polyhedron, 55
 Sensed parallels, 107
 Separation, 232ff
 Shear, 156
 Similar, 35ff
 Similarity, similitude, 153
 Simplex, 90
 Single elliptic geometry, 124ff, 251
 Special theory of relativity, 210
 Spherical,
 excess, 43
 geometry, 129ff
 triangle, 46
 Squaring a circle, 14
 SSS, 12
 Euclid I-8, 289
 Stabilizer, 186
 Stable, 133, 244
 Statistical design theory, 265
 Statistically self-similar, 215
 Steiner triple system, 275
 Subgeometry, 247ff
 Subgroup, 191
 Summit, summit angle, 113
 Symmetric,
 BIBD, 277
 group, 205
 Symmetry, 179ff, 181
 group, 182
 Synthetic, 2

 Tangent, 76, 282
 Taxicab geometry, 31, 78
 Tetrahedron, 40
 Theorem, 26
 36-officers problem, 264
 Timelike, 214
 Torus, doughnut shape, 78
 Tournament, 277

Transcendental, 16
 Transformation, 133
 group, 136
 Transformational geometry, 132ff
 Translation, 137
 Transversal, 12
 Trilinear plots, 74
 Trisecting an angle, 14

 Ultraparallels, 107
 Undefined terms, 25

 Variety, 272
 Vertex, 39

 Wallpaper pattern, 190
 Without loss of generality (WLOG), 4

 Zeno's paradox, 4

People

 Archimedes, 15
 Aristotle, 5

 Bernoulli, Jakob, 73
 Bhaskara, 7
 Bolyai, János, 100
 Bravais, Auguste, 181, 204

 Cayley, Arthur, 226, 248

 da Vinci, Leonardo, 185, 226
 Desargues, Girard, 226
 Descartes, René, 61
 Dürer, Albrecht, 226

 Einstein, Albert, 210
 Eratosthenes, 7
 Euclid, 2, 9, 226
 Eudoxus, 4
 Euler, Leonhard, 41, 265

 Fedorov, Vyatseglav, 194
 Fermat, Pierre de, 60, 67
 Fisher, Sir Ronald A., 265, 273

 Galileo Galilei, 210
 Gauss, Carl Friedrich, 17, 101

Gödel, Kurt, 25, 31

 Hausdorff, Felix, 216
 Hilbert, David, 24
 Hippocrates, 7

 Kant, Immanuel, 98
 Kepler, Johannes, 226
 Kirkman, Thomas, 265, 275
 Klein, Felix, 104, 132, 143, 226, 247

 Lie, Sophus, 132, 155

 Mandelbrot, Benoit, 216
 Minkowski, Hermann, 212
 Möbius, August, 132, 172, 226
 Monge, Gaspard, 87, 226

 Newton, Sir Isaac, 210

 Oresme, Nicole, 60

 Pascal, Blaise, 226
 Penrose, Roger, 209
 Plato, 4
 Plücker, Julius, 94, 143, 226
 Poncelet, Jean Victor, 226, 233
 Pythagoras, 3

 Riemann, Georg Bernhard, 102

 Saccheri, Girolamo, 98
 Senechal, Marjorie, 209
 Steiner, Jacob, 275

 Theaetetus, 5

 Viète, François, 60
 von Staudt, Karl, 226

 Zeno, 4

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