A

Definitions, Postulates, Common Notions, and Propositions from Book I of Euclid's *Elements*

Definitions

1. A *point* is that which has no part.
2. A *line* is breadthless length.
3. The extremities of a line are points.
4. A *straight line* is a line which lies evenly with the points on itself.
5. A *surface* is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A *plane surface* is a surface which lies evenly with the straight lines on itself.
8. A *plane angle* is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called *rectilineal*.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is *right*, and the straight line standing on the other is called *perpendicular* to that on which it stands.
11. An *obtuse angle* is an angle greater than a right angle.
12. An *acute angle* is an angle less than a right angle.
13. A *boundary* is that which is the extremity of anything.
14. A *figure* is that which is contained by any boundary or boundaries.
15. A *circle* is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.
16. And the point is called the *center* of the circle.

17. A *diameter* of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.
18. A *semicircle* is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.
19. *Rectilineal figures* are those which are contained by straight lines, *trilateral* figures being those contained by three, *quadrilateral* those contained by four, and *multilateral* those contained by more than four straight lines.
20. Of trilateral figures, an *equilateral triangle* is that which has three sides equal, an *isosceles triangle* that which has two of its sides alone equal, and a *scalene triangle* that which has its three sides unequal.
21. Further, of trilateral figures, a *right-angled triangle* is that which has a right angle, an *obtuse-angled triangle* that which has an obtuse angle, and an *acute-angled triangle* that which has its three angles acute.
22. Of quadrilateral figures, a *square* is that which is both equilateral and right-angled; an *oblong* [*rectangle*] that which is right-angled but not equilateral; a *rhombus* that which is equilateral but not right-angled; and a *rhomboid* [*parallelogram*] that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called *trapezia*.
23. *Parallel* straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

The Postulates

Let the following be postulated:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Common Notions

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.

4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

The Propositions of Book I

(Modern paraphrases are shown in brackets)

I-1. On a given finite straight line, to construct an equilateral triangle.

I-2. To place at a given point (as an extremity) a straight line equal to a given straight line. [Construct a line segment of a given length.]

I-3. Given two unequal straight lines, to cut off from the greater a straight line equal to the less. [Construct a shorter line segment on a larger one.]

I-4. [SAS] If two triangles have two sides equal to two sides, respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles, respectively, namely, those which the equal sides subtend.

I-5. In isosceles triangles, the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another. [Triangles with two congruent sides have two congruent angles.]

I-6. If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another. [Triangles with two congruent angles have two congruent sides.]

I-7. Given two straight lines constructed on a straight line (from its extremities) and meeting in a point, there cannot be constructed on the same line (from its extremities), and on the same side of it, two other straight lines meeting in another point and equal to the former two, respectively, namely, each to that which has the same extremity with it. [Given the lengths of the three sides, only one triangle can be formed in a given position.]

I-8. [SSS] If two triangles have the two sides equal to two sides, respectively, and have also the base equal to the base, they will also have the angles equal which are contained by the equal straight lines.

I-9. To bisect a given rectilineal angle.

I-10. To bisect a given finite straight line.

I-11. To draw a straight line at right angles to a given straight line from a given point on it. [Construct the perpendicular to a line from a point on a line.]

I-12. To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line. [Construct the perpendicular to a line from a point not on a line.]

I-13. If a straight line set up on a straight line make angles, it will make either two right angles or angles equal to two right angles. [The measures of supplementary angles add to 180°.]

I-14. If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal to two right angles, the two straight lines will be in a straight line with one another. [If the measures of two adjacent angles add to 180°, then the two rays not shared by the angles form a line.]

I-15. If two straight lines cut one another, they make the vertical angles equal to one another. [Vertical angles are congruent.]

I-16. In any triangle if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles. [An exterior angle of a triangle is greater than either remote interior angle.]

I-17. In any triangle two angles taken together in any manner are less than two right angles. [The measures of any two angles of a triangle add to less than 180°.]

I-18. In any triangle the greater side subtends the greater angle. [The larger side is opposite the larger angle.]

I-19. In any triangle the greater angle is subtended by the greater side. [The greater angle is opposite the greater side.]

I-20. In any triangle two sides taken together in any manner are greater than the remaining one. [Triangle inequality: The lengths of any two sides of a triangle add to more than the length of the third side.]

I-21. If on one of the sides of a triangle, from its extremities, there be constructed two straight lines meeting within the triangle, the straight lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater angle.

I-22. Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.

I-23. On a given straight line and at a point on it, to construct a rectilineal angle equal to a given rectilineal angle. [Construct a given angle.]

I-24. If two triangles have the two sides equal to two sides, respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will also have the base greater than the base.

I-25. If two triangles have the two sides equal to two sides, respectively, but have the base greater than the base, they will also have the one of the angles contained by the equal straight lines greater than the other.

I-26. [AAS and ASA] If two triangles have the two angles equal to two angles, respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle to the remaining angle.

I-27. If a straight line falling on two straight lines makes the alternate angles equal to one another, the straight lines will be parallel to one another. [For two lines cut by a transversal if alternate interior angles are congruent, the lines are parallel.]

I-28. If a straight line falling on two straight lines makes the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles, the straight lines will be parallel to one another. [For two lines cut by a transversal if corresponding angles are congruent, the lines are parallel. If the measures of the two interior angles on the same side of the transversal add to 180°, the lines are parallel.]

Note: The Parallel Postulate is needed to prove many propositions starting with I-29.

I-29. A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles. [Two parallel lines cut by a transversal have alternate interior (and exterior) angles congruent and corresponding angles congruent, and the measures of the two interior angles on the same side of the transversal add to 180°.]

I-30. Straight lines parallel to the same straight line are also parallel to one another. [If k is parallel to l and l is parallel to m, then k is parallel to m.]

I-31. Through a given point to draw a straight line parallel to a given straight line. [Construct a parallel to a given line through a given point.]

I-32. In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles. [The angle sum of a triangle is 180°. The sum of the measures of two angles equals the opposite exterior angle.]

I-33. The straight lines joining equal and parallel straight lines (at the extremities which are) in the same directions (respectively) are themselves also equal and parallel. [If $\overline{AB}$ is congruent and parallel to $\overline{CD}$ in quadrilateral $ABCD$, then $\overline{AD}$ is congruent and parallel to $\overline{BC}$.]

I-34. In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas. [In a parallelogram opposite sides are congruent, opposite angles are congruent, and the diagonals divide the parallelogram into congruent triangles.]

I-35. Parallelograms which are on the same base and in the same parallels are equal to one another. [Parallelograms with the same base and the same height have the same area.]

I-36. Parallelograms which are on equal bases and in the same parallels are equal to one another. [Parallelograms with congruent bases and heights have the same area.]

I-37. Triangles which are on the same base and in the same parallels are equal to one another. [I-35 for triangles.]

I-38. Triangles which are on equal bases and in the same parallels are equal to one another. [I-36 for triangles.]

I-39. Equal triangles which are on the same base and on the same side are also in the same parallels.

I-40. Equal triangles which are on equal bases and on the same side are also in the same parallels.

I-41. If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle. [In effect, the area formula $A = \frac{1}{2}b \cdot h$ for triangles.]

I-42. To construct, in a given rectilineal angle, a parallelogram equal to a given triangle. [Construct a parallelogram with the same area as a triangle and with a given angle.]

I-43. In any parallelogram the complements of the parallelogram about the diameter are equal to one another.

I-44. To a given straight line to apply, in a given rectilineal angle, a parallelogram equal to a given triangle. [Construct a parallelogram with the same area as a triangle, with a given side and a given angle.]

I-45. To construct, in a given rectilineal angle, a parallelogram equal to a given rectilineal figure. [Generalizes I-44 to any polygon.]

I-46. On a given straight line to describe a square. [Construct a square with a given side.]

I-47. [The Pythagorean theorem.] In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

I-48. [The converse of the Pythagorean theorem.] If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.