Plants and Human Affairs – Biol106

Surface-to-Volume Ratios in Plants

Introduction:

The purpose of this lab is to introduce you to the importance of surface-to-volume ratios (abbreviated S/V). Surface area (SA), which is expressed in squared units (***e.g.***, mm^2, μm^2), is the amount of an object that is directly exposed to the environment. For a cell, it would represent the area of the plasma membrane and for a person it would represent the amount of skin. Volume is a rough measure of the size of a structure and the amount of space it occupies. Volume is expressed by cubic units (***e.g.***, mm^3, cm^3 = milliliters). The surface-to-volume ratio (S/V) refers to the amount of surface a structure has relative to its size; or stated in a slightly more gruesome manner, S/V ratio is the amount of "skin" compared to the amount of "guts." To calculate the S/V ratio, simply divide the surface area by the volume.

The reason that surface-to-volume ratios are important is because a cell or organism continuously exchanges materials, such as food, waste, water, and heat, with its environment. Depending on the circumstances, it may be advantageous to have a small S/V while at other times a large S/V is an advantage. Thus, optimizing S/V ratios has been a driving force in the evolution of all organisms. Since S/V is a function of both size and shape, these have also been under strong evolutionary pressure.

To begin our studies we will examine the effects of both size and shape on surface-to-volume ratios. Then, we will use this information to answer other fundamental questions.

Exercise 1. The Relationship between Volume, SA, and S/V Ratio

In this exercise we will explore the mathematical relationship between volume, surface area, and the S/V ratio. Consider a cube that is one unit on a side. If it increases in size, obviously both the surface area and volume will increase. But, by how much? And, how will this affect the surface-to-volume ratio? In this exercise we will calculate the surface area, volume and s/v ratio for a series of cubes, graph the results, and then attempt to answer these questions and others. First, let's make a few predictions.

Hypothesis 1.1: As a cube gets larger, its S/V ratio will (select one: **decrease / remain the same / increase**).

Hypothesis 1.2: As a cube gets larger, the surface area of the cube will increase by *(select one: **twice / the square of / the cube of**)* the linear dimension.

Hypothesis 1.3: As a cube gets larger, the volume of the cube will increase by (select one: **three times / the square of / the cube of)** the linear dimension.

Method:

1. Complete Table 1 for a series of cubes of varying size (equations at end).

2. Prepare a graph plotting (a) Surface area vs. cube size (length of a side); (b) volume vs. cube size (length of a side); and (c) surface/volume ratio vs. cube size (length of a side). Plot all three lines on the same graph. Append your graph to your post-lab.

Table 1. Volume, surface area and surface-to-volume ratio of a series of cubes of varying sizes			
Length of a side (cm)	Surface Area (cm^2)	Volume (cm^3)	S/V
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Analysis:

1. Consider any two cubes; for example, let's use the ones that are 1 and 3 cm on a side.

    a. The 3 cm cube is ______ x larger than the smaller one.

    b. The surface area of the 3 cm cube is _____**x** larger than the smaller one. This value is: **2 times greater than the smaller one / the square of the difference in size of the cubes / the cube of the difference in size**.

    c. The volume of the 3 cube is _____x larger than the smaller one. This value is (circle): **3 times greater than the smaller one / the square of the difference in size of the cubes / the cube of the difference in size.**

2. Offer a brief explanation why the surface/volume ratio decreases as an object gets larger.

3. Consider the movie, "Honey, I Shrunk the Kids" in which Wayne, a brilliant but klutzy scientist accidentally shrinks his kids to about an inch tall. If his daughter was initially 4 foot tall and weighed 75 pounds, how much did she presumably weigh after being shrunk to one inch?

4. Did you see the classic Grade B horror movie, "The Attack of the 60 foot Woman?" How much would this woman weigh? Assume an 'average' woman who is 5 foot tall weighs 110 pounds.

5. According the *Guinness Book of World Records* the tallest living human female is 7 feet 7 ¼ inches tall. Using our assumption that a 5 foot woman weighs 100 pounds, how much do you predict this woman weighs? __________. (note: her actual weight = 462 pounds)

Exercise 2. S/V Ratios In Flattened Objects

In this exercise we will explore how flattening an object impacts the surface to volume ratio. Consider a cube made out of clay that is 8 x 8 x 8 cm on a side. Then, imagine that we can flatten this cube making it thinner and thinner while maintaining the original volume. What will happen to the surface area and s/v ratio as the cube is flattened? Let's make a hypothesis:

Hypothesis 2.1: As the cube is flattened, the s/v ratio will (*select one**: increase / decrease / remain the same***).

Method:

1. Complete Table 2. To calculate the volume of a rectangular box, multiply length by width by height. To calculate the surface area, you need to calculate the surface area of each side and then add these values together.

Table 2. Effect of flattening a box on surface/volume ratio

Box No.	Height (cm)	Length (cm)	Width (cm)	volume (cm^3)	Surface area (cm^2)	S/V ratio
1	8	8	8			
2	4	16	8			
3	2	16	16			
4	1	32	16			
5	0.5	32	32			

Exercise 3. S/V Ratios in Elongated Objects

In this exercise we will explore how elongating an object impacts the surface to volume ratio. Once again we'll start with a cube of clay that is 8 x 8 x 8 cm on a side. Now, imagine that we pull on the ends to make it longer and longer while maintaining the original volume. What will happen to the surface area and s/v ratio as the box is elongated?

Hypothesis 3.1: As the cube is elongated the s/v ratio will *(select one: **increase / decrease / remain the same**).*

Method:

1. Complete Table 3.

Table 3. Effect of elongating a box on surface/volume ratio						
Box No.	Height (cm)	Length (cm)	Width (cm)	volume (cm^3)	Surface area (cm^2)	S/V ratio
1	8	8	8			
2	16	4	8			
3	32	4	4			
4	64	2	4			
5	128	2	2			

Exercise 4. Shape and S/V Ratio

In this exercise we will explore the impact of shape on surface to volume ratios. Consider three objects – a sphere (or ball), a cube, and a long, skinny box (or filament) – all of which have the same volume (1.0 cm). Which object will have the greatest S/V ratio? Which object is in contact with a greater portion of its environment? One way to answer the latter question is to calculate the amount or volume of the environment that is within 1.0 cm of the object. This is important since all organisms/structures exchange materials with their environment and the exchange occurs between the organism and the environment directly in contact with the object.

Hypothesis 4.1. The shape with the largest S/V ratio will be the (*select one:* ***cube / sphere / filament***)

Hypothesis 4.2. The shape that is exposed to the greatest amount of its environment is the (*select one*: ***cube / sphere / filament***)

Method:

1. Calculate the volume, surface area and S/V ratio for each of three different-shaped objects in Table 4.

2. To calculate the volume of environment within 1.0 cm of each object, imagine that the object is surrounded by a larger object that is 1.0 cm larger on all sides. Calculate the volume of this larger object and then subtract the volume of the smaller shape.

Table 4. Effect of shape on surface-to-volume ratios					
Shape	Dimensions (cm)	Volume (cm^3)	Surface Area (cm^2)	S/V ratio	Volume of environment within 1.0 cm
Sphere	1.2 diameter	1.0			
Cube	1 x 1 x1	1.0			
Filament	0.1 x 0.1 x 100	1.0			

Analysis:

1. On a separate sheet of paper (preferably graph paper), make a sketch, ***to scale***, of the three objects (sphere, cube, filament).

2. Which shape is better for maximizing a structure exchanging materials with its environment: a **sphere or filament**? Which shape is better for minimizing exchange with the environment: a **sphere or filament**?

3. Explain why plants are essentially "filamentous" and animals are "spherical".

Exercise 5. Measuring S/V ratios of different plants

Considering the theoretical calculations we made above, we hypothesize that plants that live in a xeric (dry) environment will have a smaller S/V than a plant that lives in a mesic (moderate) environment. You will be provided with the leaves of two plants (Jade plant (*Crassula argenta*) and honeysuckle (*Lonicera tartarica*) or other species. Examine the leaves and then make some predictions about the conditions in which these species evolved.

Hypothesis 5.1. The leaf of the (, fill in) has the greatest S/V ratio.

Hypothesis 5.2. The plant best adapted for xeric conditions is: (fill in).

Method.

1. Estimate the SA of each leaf by tracing the leaf on graph paper. Count the number of boxes completely within the tracing (Table 5). Count as 0.5 any box that is intersected by the tracing. Measure the size of one box (_____ cm x ______ = _____ cm^2). Determine the surface area of one side of a leaf. Multiply by 2 for both sides of the leaf. Note: we will ignore the surface area of the edge of the leaves.

2. Calculate the volume by weighing each leaf to the nearest 0.01 g. This will give a rough approximation to volume since the fresh weight of the leaves is largely water (density = 1 gm/cm^3).

3. Calculate S/V and complete table 5.

Table 5. S/V ratio of leaves of different plants.

Species	Number of boxes	SA (cm^2) – one side of leaf	Ttl SA (cm^2) – both sides	volume (gm = cm^3)	S/V
Jade plant					
Honeysuckle					

Analysis:

1. Do our data match our predictions?

Exercise 6: Water loss and S/V ratios

Water availability has been a major factor directing the evolution of leaf shape in plants. We've hypothesized that a larger S/V ratio will result in greater water loss. In this exercise we will test this hypothesis by using sponges that will serve as a model leaf. You will be given a large sponge and a smaller sponge to serve as models of plant leaves.

Hypothesis 6.1. In a given period of time, the greatest total amount of water will be lost from the (**large sponge / small sponge / both sponges will loose an equal amount of water**).

Hypothesis 6. 2. In a given period of time, the sponge that will loose the greatest percentage of its initial water content is the (**large sponge / small sponge / both sponges will loose an equal amount of water**).

Hypothesis 6.3. Per unit surface area, the sponge that will loose the greatest amount of water is the: (**large sponge / small sponge / both sponges will loose an equal amount of water**).

<u>Method</u>:

2. Measure the size of each sponge and then calculate SA, volume and S/V ratio. Complete the appropriate rows in Table 6.
3. Soak the sponge and then squeeze out the excess.
4. Weigh sponge.
5. Allow the sponge to sit on a rack for about an hour. Then reweigh.
6. Complete the calculations in Table 6.

Table 6. Water loss from leaf models

	large sponge	small sponge
length (cm)		
width (cm)		
height (cm)		
SA (cm^2)		
vol (cm^3)		
S/V		
Initial weight sponge (g)		
Final weight sponge, after evaporation (g)		
Amount of water lost (g = mL)		
Water loss per unit volume (g / cm^3)		
Water loss per unit surface area (g / cm^2)		

1. Explain why plants in a rain forest typically have large, thin leaves, while cacti have no leaves at all.

2. An orchid specimen is available to look at. This plant lives in as an epiphyte in a rain forest. Explain why its leaves are thick and leathery.

Exercise 7. Why Are Cells Small?

The typical eukaryotic cell is rather small, approximately 100 µm in diameter. With a few exceptions, the cells of all organisms are about this size. Thus, the reason that an elephant is bigger than a mouse, or a redwood is bigger than a dandelion, is because redwoods and elephants they have ***more*** cells, not larger ones. So, why are the cells so small?

In this exercise we will use a cylinder of agar to serve as a model cell. The agar has an acid-sensitive dye called bromocresol green incorporated into it. The dye turns from blue to yellow (clear) in the presence of acid. To simulate the cell feeding or obtaining a critical nutrient from its environment, we will place the agar cylinder in a container of vinegar and monitor the color of the cylinder. The uptake of acid, and hence colorless areas of the cell model, will represent the uptake of food/nutrients by the cell. From this, we can calculate the percent of each cell that was fed during the incubation period.

Hypothesis 7.1: The cell model in which the greatest percentage of its volume will get fed is the (*select one: small / large*) one.

Hypothesis 7.2: The cell model in the greatest danger of starving is the (*select one: small / large*) one.

Methods:

1. Obtain 2 agar cylinders (cell models), one small and one large. Wearing gloves, measure the length (height) and diameter of each cell and record your data in Table 7.

2. Using a plastic spoon, carefully place each cylinder in a container containing vinegar (be careful).

3. Allow the cylinders to sit in the vinegar for a few minutes until most of the blue color is gone from the smallest cylinder. Then, remove both models with the plastic spoon and place them on a piece of paper towel.

4. Measure the height and diameter of the colored areas remaining in each model. Record these data in table 5.

5. Complete the calculations.

Table 7. Volume, surface area and surface-to-volume ratio of a series of cubes of varying sizes

		small cell	large cell
Colored portion before feeding	dia (mm)		
	radius (mm)		
	height (mm)		
	SA (mm^2)		
	vol (mm^3)		
	S/V		
Colored portion after feeding	radius (mm)		
	height (mm)		
	SA (mm^2)		
	vol (mm^3)		
	S/V		
Percent of volume (cell) fed = (initial volume - final volume)/initial volume x 100			

Analysis

1. Append a copy of Table 5 showing completed calculations.

2. Fill-in-the-blank with the appropriate response.
   a. The cell model in which cell the greatest portion will get fed is the _______ one.
   b. The cell model in the greatest danger of starving is the ___________ one.

3. Suggest a reason why the rate of cell growth slows as a cell gets larger.

4. Suggest a reason why cells divide when they get large.

Exercise 8. Why Do Mice Have Greater Metabolic Rates Than Elephants?

It is well known that there is an inverse relationship between body size and metabolic rate. That is, the larger the animal the lower its basal rate of metabolism. Therefore, mice have a greater metabolic rate than elephants. The purpose of this exercise is to determine the reason for this relationship. We will use beakers of water to represent organisms with the same shape but different size. First let's make a few predictions:

Hypothesis 6.1. The (*select one: small / large*) beaker will loose the most total heat.

Hypothesis 6.2. The (*select one:* ***small / large***) beaker will loose the heat at the greatest rate.

Hypothesis 6.3. The (*select one:* ***small / large***) beaker would require the most total heat input to maintain a constant temperature.

Hypothesis 6.4. The (*select one:* ***small / large***) beaker would require the greatest total heat input relative to its size to maintain a constant temperature.

Method:

1. Obtain an aluminum pan in which 30 and 100 mL beakers are glued to the bottom. Pipet 8 mL of water into the small beaker. Then using a ruler, measure in centimeters the height and width of the water column in the beaker and record your data in Table 8 (rows 1 & 2).

2. Using a graduate cylinder, put 80 mL of water into the medium-sized beaker and measure the height and width of the water column. Record your data.

3. Empty the water from the beakers and then pack ice around the empty containers in the pan to a depth of about two inches. Then add cold water to bring the water level up to the top of the ice.

4. Measure 80 mL of hot water from the hot plate **(be careful!)** with a graduate cylinder and place it into the 100 mL beaker.

5. Place the thermometer in the hot water. At first the temperature will rise. When it begins to fall again, measure the temperature. If it is below 40 C, start again with hotter water.

6. Slowly stir the water. When the temperature reaches 40 C, record the time. Exactly two minutes later record the temperature again. Record your data in Table 6, row 9.

7. Repeat steps 4 - 6 except pipet 8 mL of water into the smaller container.

Data & Analysis: *To analyze our data, complete the following steps.*

1. Calculate the radius of each container (row 3)

2. Calculate the surface area of the water column in each beaker using the equation for a cylinder (row 5). Although we could use our measurements of water column height and width to calculate the volume of each container (row 4), we don't need to do so since we've already measured the volume of water in each with a graduate cylinder.

3. Calculate the surface/volume ratio (row 6)

4. Calculate the initial and final heat content (rows 8 and 10) by multiplying the temperature (rows 7 and 9) by the volume of water (row 4) by the specific heat of water (4.2 joules/cm^3 C). [Note - Specific heat is an indication of the amount of heat it takes to change the temperature of a substance. Joules are a standard energy unit.] Thus, this equation is:
   Heat content (joules) = temp. (C) x vol. (cm^3) x specific heat of water (4.2 joules/cm^3 C)

5. Calculate the total heat loss during the two-minute experiment (row 11) by subtracting the final heat content (row 10) from the initial heat content (row 8).

6. Calculate the percent of heat lost by each container (row 12) by the following equation: (initial heat content - final heat content)/initial heat content x 100

7. Calculate the relative heat loss (joules lost/cm^3) for each volume (row 13) by dividing the total heat loss (row 11) by the volume of the vessel (row 4).

8. Calculate the relative heat loss (joules lost/cm^3) for each volume (row 13) by dividing the total heat loss by the volume of the vessel (row 4).

9. Calculate the percent heat lost (row 14).

Table 8. Effect of size on the rate of cooling (equations)		
	small	large
1. height (cm)		
2. diameter (cm)		
3. radius (cm)		
4. volume (cm^3)	8	80
5. surface area (cm^2)		
6. S/V ratio		
7. initial temp (C)	40	40
8. initial heat content (J)		
9. final temp (C)		
10. final heat content (J)		
11. total heat loss (Joules)		
12. total heat loss (Joules/min)		
13. relative heat loss (J/cm^3)		
14. Percent heat loss [(initial heat content - final heat content)/initial heat content * 100]		

Analysis:

1. Fill-in-the-blank with the appropriate response.
    a. The ___________ beaker lost the most total heat.
    b. The ___________ beaker lost heat at the greatest rate
    c. The ___________ beaker would require the most total heat input to maintain a constant temperature.
    d. The ___________ beaker would require the greatest total heat input relative to its size to maintain a constant temperature.
2. Answer the following questions either "mouse" or "elephant."
    a. Looses the most heat during a given period of time ___________
    b. Looses the most heat relative to its size during a given time period
    c. Needs to eat the most food _____________
    d. Eats the most food relative to its size _______________
    e. Referring to your data, explain why mice have a greater metabolic rate than an elephant.

Exercise 9. Biological Applications of S/V

Although these exercises are relatively simple, they provide the basis to explain many biological observations. Listed below are a just a few questions that can be answered using S/V reasoning.

1. Explain why small animals have a higher metabolic rate than large animals.
2. Explain why shrews are voracious feeders.
3. Explain why cats can fall off tall buildings and survive. Why do people splat?
4. Explain the advantages/disadvantages of block vs. cube ice.
5. Describe the scientific inaccuracy in the episode of Goldilocks and the porridge.
6. Medieval churches were often built in the shape of a crucifix. Explain why.
7. The earth is geologically active (has a molten core) but the moon is apparently no longer geologically active. Explain why using S/V ratios.
8. Explain why the cells of the spongy layer of a plant leaf are irregularly shaped.

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Useful Equations:

Useful Equations		
Shape	**Equation for Volume**	**Equation for Surface Area**
cube	L x W x H	L x W x 6
box (filament)	L x W x H	determine SA for each side then add
sphere	$4/3\ (\pi)\ r^3 = 4.189\ r^3$	$4\ (\pi)\ r^2 = 12.57\ r^2$
cylinder	$\pi\ r^2\ H$	$2\ \pi\ r^2 + 2\ \pi\ r\ H = 2\ \pi\ r\ (r + H)$
circle		$\pi\ r^2$
square or rectangle		L x W